Shape Sensitivity Analysis and Optimization of Current-Carrying Conductor for Current Distribution Control

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This paper proposes a sensitivity analysis for shape optimization of the current-carrying conductor. The 3 dimensional sensitivity formula is derived using the material derivative concept and the adjoint variable method. The objective function is defined in the conductor region and the design variable is on the conductor boundary, where the homogeneous Neumann condition is applied. The deformation of the conductor shape is expressed using the level set method. Two numerical examples are tested to show usefulness of the proposed method.

***Index Terms*—Shape sensitivity, Current-carrying conductor, Finite element method, Level set method, Shape optimization**

I. Introduction

CONDUCTOR shape design problems exist in various systems or devices such as power cable junction, current distribution divider, fault current protection system, IC chip inductor, IC leads, etc. For example, the large fault current in a power protection system should be evenly distributed inside the current-carrying conductor to endure thermal melting or damage. The IC current leads and the IC chip inductor should be designed to minimize thermal concentration for their reliable operation.

Until now, however, the shape design of those current-carrying conductor has been dependent mainly on the designer's experience or the rule of trial and error. So, their systematic and accurate design for improved performance requires a new design method that is based on the electromagnetic field analysis.

When the conductor size such as width is small or the eddy current effect is negligible at a given frequency, the current distribution in the conductor is determined only by the conductor shape. The current distribution in the conductor is modelled as the Laplace equation for the potential variable, which comes from the continuity equation in the steady state. The Dirichlet boundary condition is imposed on the surfaces of external voltage connections, and the homogeneous Neumann boundary condition is applied on the remaining conductor surfaces since the current density has no normal components. Therefore, the shape design problem of current-carrying conductor problem is the design problem of the Neumann boundary.

In this paper, the 3 dimensional shape sensitivity formula is analyticall derived in a closed form using the material derivative concept and the adjoint variable technique, which are based on the continous, not discrete, variational equations [1]. The objective function is defined as a general function of the potential or the electric field in the conductor region. The level set method is employed to express the evolving shapes. The velocity field in the level set equation is coupled with the shape sensitivity formula, and the coupled equation is solved to provide with the optimal shape. The state and adjoint variable are numerically calculated with the finite element method. Two numerical examples are tested to demonstrate feasibility and usefulness of the proposed method.

II. Continuum sensitivity analysis

In an electrostatic system shown in Fig.1, let Ω be a spatial domain surrounded by the outer boundary Γ . The boundary consists of Γ^0 and Γ^1 where the Dirichlet and homogeneous Neumann conditions are applied, respectively. $\hat{\boldsymbol{n}}$ is the outward unit normal vector, σ is the electrical conductivity, ε_0 is the permittivity, m_p is the characteristic function that represents a region where the objective function is defined, The design variable is the Neumann boundary in this problem.

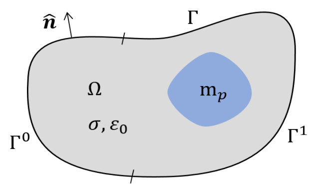


Fig. 1. Conceptual diagram of electrostatic system

The variational governing equation is the same to the electrostatic system as:

$$a(V,\bar{V}) \equiv \int_\Omega \varepsilon_0 \nabla V \cdot \nabla \bar{V}\, d\Omega = \int_\Omega \rho \bar{V}\, d\Omega \equiv l(\bar{V}) \quad \forall\, \bar{V} \in \Phi \tag{1}$$

where Φ is the space of admissible function, V is the electric potential, which is the state variable, and ρ is the electric charge density. The objective function is a regional integral as follow:

$$F = \int_\Omega g(V, \nabla V)\, m_p\, d\Omega \tag{2}$$

where g is a differentiable function. An augmented objective function G is introduced using (1) and (2).

$$G = F + l(\bar{V}) - a(V,\bar{V}) \quad \forall\, \bar{V} \in \Phi \tag{3}$$

The sensitivity formula is analytically derived using material derivative of the augmented objective function and adjoint variable method [1].

$$\dot{G} = -\int_{\Gamma^0} \varepsilon_0 E_t(V)\, E_t(\lambda)\, V_n\, d\Gamma \tag{4}$$

where E_t is the tangential component of electric field on the design variable, λ is the adjoint variable, and V_n is the normal component of the velocity field which contributes to the shape deformation. The sensitivity formula is expressed as a boundary integral over the design variable. Both V and λ of the variational equations are calculated by using finite element method [2].

The velocity of the design variable is calculated as follow:

$$V_n = k\,E_t(V)E_t(\lambda) \tag{5}$$

where k can be positive or negative for maximization or minimization problem, respectively. The level set method is utilized to conveniently express the shape. The velocity (5) is substituted for the velocity term in the level set equation [3].

III. Numerical Examples

Two shape optimization examples are tested using the continuum sensitivity formula in two dimensional space. The design goal is to minimize electrical resistance of a conductor maintaining the conductor volume during the optimization. Since the source of the system is electric voltage, minimizing resistance is equivalent to maximizing system energy. The system energy W of current-carrying conductor is expressed in form of electric field energy:

$$W = \int_{\Omega} \sigma \mathbf{E}(V)\cdot\mathbf{E}(V)\,d\Omega = \frac{2\sigma}{\varepsilon_0}\int_{\Omega}\frac{\varepsilon_0}{2}\mathbf{E}(V)\cdot\mathbf{E}(V)\,d\Omega \tag{6}$$

Thus, the electric field energy is taken as the objective function. In this case, the adjoint variable is same as the state variable.

The following two examples consist of two fixed conductors and a connection conductor. Only the connection conductor is the design object to be optimized.

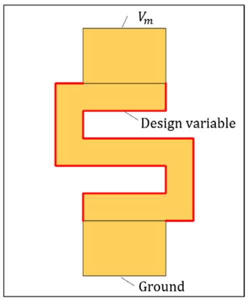
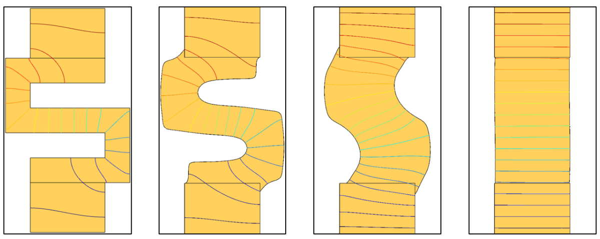


Fig. 2. Initial design: example 1

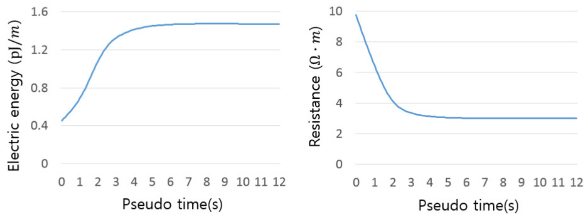
The first example is tested to show feasibility of the derived sensitivity formula. Fig. 2 shows the initial design of shape optimization. In this problem, the analytical solution is known, and we can expect that the resistance is minimized when the connection part is a rectangle.



0 s 1 s 3 s 12 s

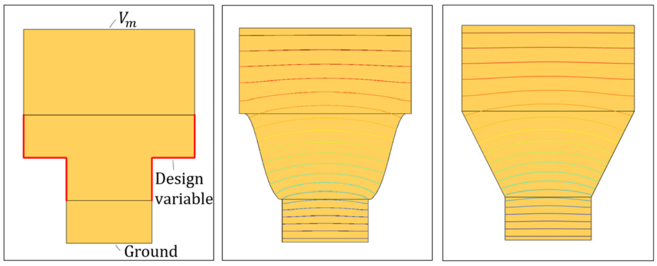
Fig. 3. Optimization processes: example 1

Fig. 3 shows shape deformations during the design process. As expected, the final shape in 12 seconds is obtained and the objective function variation during the optimization procedure is shown in Fig. 4(a). Also, Fig. 4(b) shows variation of resistance when σ = 1 [S/m].



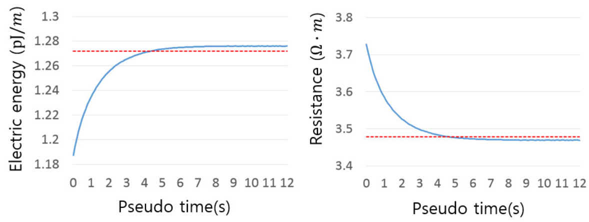
(a) (b)

Fig. 4. Variation during optimization (a) Objective function (b) Resistance



(a) (b) (c)

Fig. 5. Example 2. (a) Initial design. (b) Final design. (c) Straight line design



(a) (b)

Fig. 6. Variation during optimization (a) Objective function (b) Resistance

The proposed method is applied to the second design example that two different size conductors are connected. Fig. 5(a) shows the initial shape of the example. In Fig. 6, the blue solid and the red dotted lines represent variation of the objective function and resistance of Fig. 5(b) and (c), respectively. Fig. 5(b) is the final shape where the objective function is converged. It is worth noting that the final shape has smooth curved lines but not straight lines. These results shows usability of the proposed optimization method.

References

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